Practice Makes Perfect Understanding

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| **Problem 1:**  *f* is a linear function. Values of x and f(x) are given in the table below; complete the table.   |  |  | | --- | --- | | *x* | *f(x)* | | *-3* | *17* | | *0* | *--* | | *--* | *1* | | *4* | *-18* | | *7* | *--* | | *--* | *-30* | |

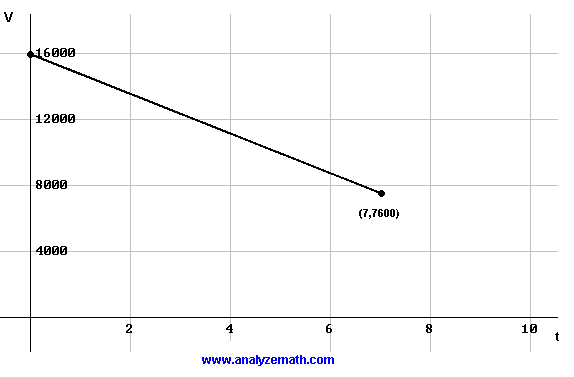
**Problem 2:**   
A family of linear functions is given by

*f(x) = mx + (3 - 2 m)*

where *x* is the independent variable and *m* is a constant.   
a) Graph *f* for *m = 0, 1, 2, -3* and *-5*   
b) What do all the graphs in part a) have in common?   
c) Justify your answer to part b) analytically.   
d) Write the equation of the family of functions whose graphs pass by the same point *(- 2 , - 4).*

**Problem 3:**   
A high school had 1200 students enrolled in 2003 and 1500 students in 2006. If the student population P; grows as a linear function of time t, where t is the number of years after 2003.   
a) How many students will be enrolled in the school in 2010?   
b) Find a linear function that relates the student population to the time t.

**Problem 4:**   
The graph shown below is that of the linear function that relates the value V (in $) of a car to its age t, where t is the number of years after 2000.

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a) Find the slope and interpret it.   
b) What will be the value of the car in the year 2010?

**Answer sheet**

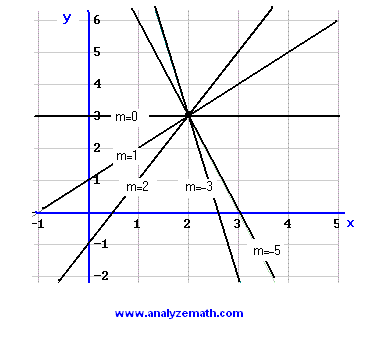
**Solution to Problem 1:**

* *f* is a linear function whose formula has the form   
    
  *f(x) = a x + b*
* where a and b are constants to be found. Note that 2 ordered pairs (-3,17) and (4,-18) are given in the table. These two ordered pairs are used to write a system of linear equations as follows   
    
  17 = - 3 a + b and -18 = 4 a + b
* Solve the above system to obtain a = - 5 and b = 2 and write the formula for function *f* as follows   
    
  f(x) = - 5 x + 2
* We now use the formula for f to find f(x) given x or find x given f(x).   
    
  for x = 0 , f(0) = -5(0) + 2 = 2   
    
  for f(x) = 1 , 1 = -5 x + 2 which gives x = 1 / 5   
    
  for x = 7 , f(7) = -5(7) + 2 = - 33   
    
  for f(x) = - 30 , -30 = -5 x + 2 which gives x = 32 / 5
* We now put the values calculated above in the table.

|  |  |
| --- | --- |
| *x* | *f(x)* |
| *-3* | *17* |
| *0* | *2* |
| *1 / 5* | *1* |
| *4* | *-18* |
| *7* | *-33* |
| *32 / 5* | *-30* |

**Solution to Problem 2:**

* a)

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* b) All the graphs pass by the same point (2 , 3)
* c) To prove that all lines described by the equation *f(x) = mx + (3 - 2 m)* pass by the point (2 , 3), show that f(2) = 3   
    
  f(2) = 2 m + (3 - 2m) = 3
* d) The point slope form of the equation of a line is used to find equation of the family of lines that pass by the point (-2,-4) is found as follows   
    
  y - (-4) = m (x - (-2))   
    
  y = mx + (2m - 4)
* As an exercise, graph the above equation for different values of m and check that all the lines obtained pass by the point (-2 , -4)

**Solution to Problem 3:**

* a) The given information may be written as ordered pairs (t , P). The year 2003 correspond to t = 0 and the year 2006 corresponds to t = 3, hence the 2 ordered pairs   
    
  (0, 1200) and (3, 1500)
* Since the popopulation grows linearly with the time t, we use the two ordered pairs to find the slope m of the graph of P as follows   
    
  m = (1500 - 1200) / (6 - 3) = 100 students / year
* The slope m = 100 means that the students population grows by 100 students every year. From 2003 to 2010 there are 7 years and the students population in 2010 will be   
    
  P(2010) = P(2003) + 7 \* 100 = 1200 + 700 = 1900 students.
* b) We know the slope and two points, we may use the point slope form to find an equation for the population P as a function of t as follows   
    
  P - P1 = m (t - t1)   
    
  P - 1200 = 100 (t - 0)   
    
  P = 100 t + 1200

**Solution to Problem 4:**

* a) Find two points from the graph   
    
  (0 , 16000) and (7 , 7600)
* Use the above points to find the slope m   
    
  m = (7600 - 16000) / (7 - 0) = - 1200 $ / year
* A slope of - 1200 $ / year means that the value of the car decreases by 1200 $ every year.
* b) In 2010, t = 10. There are 3 years from t = 7 to t = 10. The value of the car will be given by   
    
  7600 - 3 \* 1200 = 4000 $

**http://www.analyzemath.com/math\_problems/linear\_func\_problems.html**